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Technical Memorandum

A FORTRAN COMPUTER PROGRAM TO EVALUATE AND PLOT THE STATISTICS OF THE MAGNITUDE-SQUARED COHERENCE FUNCTION

Date: 5 February 1986

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ABSTRACT

This document contains computer listings for two FORTRAN computer programs. The first calculates the probability density function associated with the estimated magnitude-squared coherence function and the second calculates the cumulative distribution function associated with the estimated magnitude-squared coherence function.

ADMIMISTRATIVE INFORMATION

This memorandum was prepared by Helen A. Duling, code 33142, Systems Analysis and Applications Group.
The author of the memorandum is located at the New London Laboratory, Naval Underwater Systems Center, New London, Connecticut 06320.

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Dr. G. Clifford Carter of code 3314 is responsible for providing me with the necessary documents and information which enabled me to investigate and calculate the cumulative distribution and probability density functions associated with the estimation of the magnitude-squared coherence function.

I. INTRODUCTION

The purpose of this memorandum is to document the FORTRAN code used to calculate the probability density function and the cumulative distribution function associated with the estimated mean-squared coherence function, and to define the variables used in calculation.

II. CALCULATIONS

The first order probability density function (PDF) for the estimate of the magnitude-squared coherence function (MSC) given the true value of the MSC and the number of independent segments processed, n, can be written as (equation 4.4 in (1))

$$P(|\hat{\gamma}|^2 | n, |\gamma|^2) = (n-1)\left[\frac{(1-|\gamma|^2)(1-|\hat{\gamma}|^2)}{(1-|\gamma|^2|\hat{\gamma}|^2)^2}\right]^n \cdot \frac{(1-|\gamma|^2|\hat{\gamma}|^2)}{(1-|\hat{\gamma}|^2)^2} \cdot {}_2F_1(1-n, 1-n; 1; |\gamma|^2|\hat{\gamma}|^2)$$

where

$|\hat{\gamma}|^2$ = estimated MSC = ESTCOH

$|\gamma|^2$ = actual MSC = COH

${}_2F_1(-m, b; c; z)$ = hypergeometric function

Note F (-m,b;c;z) can be expressed as (equation 15.4.1 in (2))

$${}_2F_1(-m, b; c; z) = \sum_{n=0}^{m} \frac{(-m)_n\,(b_n)}{(c_n)} \cdot \frac{z^n}{n!}$$

The first-order cumulative distribution function for the estimate of the MSC, given the true value of MSC and the number, n, of independent segments processed, can be written as (equation 4.6 in (1)),

$$P(|\hat{\gamma}|^2 / n, |\gamma|^2) = |\hat{\gamma}|^2 \cdot \left[\left(\frac{1-|\gamma|^2}{1-|\gamma|^2|\hat{\gamma}|^2} \right)^n \cdot \sum_{k=0}^{n-2} \left(\frac{1-|\hat{\gamma}|^2}{1-|\gamma|^2|\hat{\gamma}|^2} \right)^k \right]$$

$$\cdot {}_2F_1(-k, 1-n; 1; |\gamma|^2|\hat{\gamma}|^2)$$

Note that the hypergeometric function F (-l,1-N;1;z) can be expressed as (equation 8 in (3)),

$${}_2F_1(-l, 1-N; 1; z) = \sum_{k=0}^{l} T_k$$

where

$$T_k = \frac{(-l)_k (1-N)_k z^k}{(1)_k \, k!}$$

$$T_0 = 1$$

$$\frac{T_k}{T_{k-1}} = \frac{(k-1-l)(k-1+1-N) \cdot z}{k^2}$$

III. EXAMPLE CALCULATION

The following is an example calculation corresponding to the specific values :

$$MSC = 0.3$$
$$n = 32$$

The outputs of the FORTRAN programs are tabulated in Tables 1 and 2, and shown graphically in Figures 1 and 2.

```
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
c                                                            c
c PROBABILITY DENSITY FUNCTION Verification                  c
c file : PDF.FOR                                             c
c Helen A. Duling                                            c
c                                                            c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
c NUSC                                                       c
c Scientific and Engineering Studies                         c
c Coherence Estimation                                       c
c                                                            c
c Purpose : The purpose of this program is to verify the     c
c Probability Density Function found on page 28., TR4343     c
c Estimation of the Magnitude Squared Coherence Function     c
c (Spectrum) by G.C.Carter . The PDF is calculated given     c
c n = 32, magnitude squared coherence function = 0.3 .       c
c The estimated c value ( magnitude squared  coherence       c
c function ) varies from 0 to 1.                             c
c                                                            c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
         REAL PDF,TEMP1,TEMP2,COH
         REAL Z,HSUM,ESTCOH,TERM4
         INTEGER N,I,TERM1,TERM2,L

         OPEN (UNIT=3, FILE='VERIFY2',STATUS='NEW')

cccccccccccccccccccccccccccccccccccc
c Initialization - set constants  c
cccccccccccccccccccccccccccccccccccc

         N       = 32
         COH     = 0.3
         ESTCOH  = 0.0
         TEMP1   = 0.0
         TEMP2   = 0.0
         PDF     = 0.0
cccccccccccccccccccccccccccccccccccc
c Format statements               c
cccccccccccccccccccccccccccccccccccc

   1       FORMAT (10x,a)
   2       FORMAT (1x)
   3       FORMAT (20x,a,7x,a)
   4       FORMAT (22x,F7.5,9x,F7.5)
   5       FORMAT (20x,a)
   6       FORMAT (20X,A,1X,I2,A,1X,F2.1)

        WRITE (3,1)  'Probability Density Function Calculation'
        WRITE (3,2)
        WRITE (3,6)  'Given : n = ',N,' and  c = ',COH
        WRITE (3,2)
        WRITE (3,3)  'c estimate', 'PDF value'
        WRITE (3,2)
        WRITE (3,3)  '__________','_________'
        WRITE (3,2)
```

```
c ESTCOH is varied from 0 through 1 in increments of 0.025

        DO  I=1,40,1
        ESTCOH = (I*0.025)
        TEMP1 = ((((1-COH)*(1-ESTCOH))/((1-(ESTCOH*COH))**2))**N)
        TEMP2 = ((1-(COH*ESTCOH))/((1-COH)**2))
        TERM1 = N-1
        TERM2 = 1-N
        TERM4 = COH*ESTCOH

        CALL HYPER (TERM1,TERM2,TERM4,HSUM)
        PDF = ((N-1)* TEMP1 * TEMP2 * HSUM)

        WRITE (3,4) ESTCOH,PDF
        WRITE (3,2)
        PDF   = 0.0
        TEMP1 = 0.0
        TEMP2 = 0.0
        END DO

        END

 C SUBROUTINE - computes the hypergeometric function

       SUBROUTINE HYPER (L, TERM2,Z,HSUM)

       INTEGER L,K,TERM2
       REAL T

       HSUM = 0.0
       T=0.0
       DO K = 0,L
          IF (K .EQ. 0) THEN
                T = 1.0
             ELSE
                T = (((K-1-L)*(K-1+TERM2)*Z)*T)/(K**2)
          ENDIF

       HSUM = HSUM + T
       END DO

                                                   RETURN

       END
```

```
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
c                                                            c
c CUMULATIVE DISTRIBUTION FUNCTION Verification               c
c file : CDF.FOR                                             c
c Helen A. Duling                                            c
c                                                            c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
C NUSC                                                       c
c Scientific and Engineering Studies                         c
C Coherence Estimation                                       c
c                                                            c
c Purpose : The purpose of this program is to verify the     c
c Cumulative Distribution Function found on page 33. of      c
c TR4343 Estimation of the Magnitude Squared Coherence       c
c Function (Spectrum) by G. Clifford Carter.                 c
c The PDF is calculated given n = 32, and magnitude squared c
c coherence function = 0.3                                   c
c The estimated c value (magnitude squared  coherence        c
c function) varies from 0 to 1.                              c
c                                                            c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

        REAL CDF, TEMP, SUM, COH, ESTCOH,HSUM
        REAL PARTIAL, Z,TERM4
        INTEGER N, K, KK, I,L,TERM1,TERM2

        OPEN (UNIT=3, FILE = 'VERIFY1',status='new')

cccccccccccccccccccccccccccccccccccc
c Initialization - set constants  c
cccccccccccccccccccccccccccccccccccc
        N      = 32
        COH    = 0.3
        ESTCOH = 0.0
        TEMP   = 0.0
        PARTIAL = 0.0
        CDF     = 0.0
cccccccccccccccccccccccccccccccccccc
c Format statements               c
cccccccccccccccccccccccccccccccccccc

1       FORMAT(10x,a)
2       FORMAT(1x)
3       FORMAT(20x,a,9x,a)
4       FORMAT(22x,F7.5,1x,F18.5)
5       FORMAT(20x,a)
6       FORMAT(20X,A,1X,I2,A,1X,F2.1)

        WRITE(3,1) 'Cumulative Distribution Function Calculation'
        WRITE(3,2)
        WRITE(3,6) 'Given : n = ',N,' and  c = ',COH
        WRITE(3,2)
        WRITE(3,3) 'c estimate', 'CDF value'
        WRITE(3,2)
        WRITE(3,3) '__________','_________'
        WRITE(3,2)
```

```
c ESTCOH is varied from 0 through 1 in increments of 0.025

DO  I=1,40,1
       ESTCOH = (I*0.025)
       TEMP = ((( 1-COH)/(1-(ESTCOH * COH ))) ** N )
 DO  KK=1,N-1
                                                    K=KK-1
    SUM = 1.0
    IF (ESTCOH .NE. 1.0) THEN
        SUM = ( ((1.0-ESTCOH)/(1.0 - (COH * ESTCOH))) ** K )
       ENDIF

    TERM1 = K
    TERM2 = 1-N
    TERM4 = COH*ESTCOH
    CALL HYPER(TERM1,TERM2,TERM4,HSUM)
    PARTIAL = PARTIAL + (SUM  * HSUM)
  END DO
  IF (ESTCOH .EQ. 1.0) THEN
      CDF = 1.0
      ELSE
      CDF = (ESTCOH * TEMP * PARTIAL)
   ENDIF

   WRITE (3,4) ESTCOH,CDF
   WRITE (3,2)
   CDF     = 0.0
   PARTIAL = 0.0
   TEMP    = 0.0

   END DO
   END

C SUBROUTINE - computes the hypergeometric function

       SUBROUTINE HYPER (L, TERM2,Z,HSUM)

       INTEGER TERM2,K,L
       REAL T

       HSUM = 0.0
       T    = 0.0
       DO K = 0,L
          IF (K .EQ. 0) THEN
             T = 1.0
             ELSE
             T = (((K-1-L)*(K-1+TERM2)*Z)*T)/ (K**2)
          ENDIF
       HSUM = HSUM + T
       END DO

       RETURN
       END
```

Probability Density Function Calculation

Given : n = 32 and c = .3

c estimate	PDF value
0.05000	0.07430
0.10000	0.46916
0.15000	1.41735
0.20000	2.76195
0.25000	3.87229
0.30000	4.09393
0.35000	3.32142
0.40000	2.06627
0.45000	0.97083
0.50000	0.33455
0.55000	0.08077
0.60000	0.01276
0.65000	0.00120
0.70000	0.00006
0.75000	0.00000
0.80000	0.00000
0.85000	0.00000
0.90000	0.00000
0.95000	0.00000
1.00000	0.00000

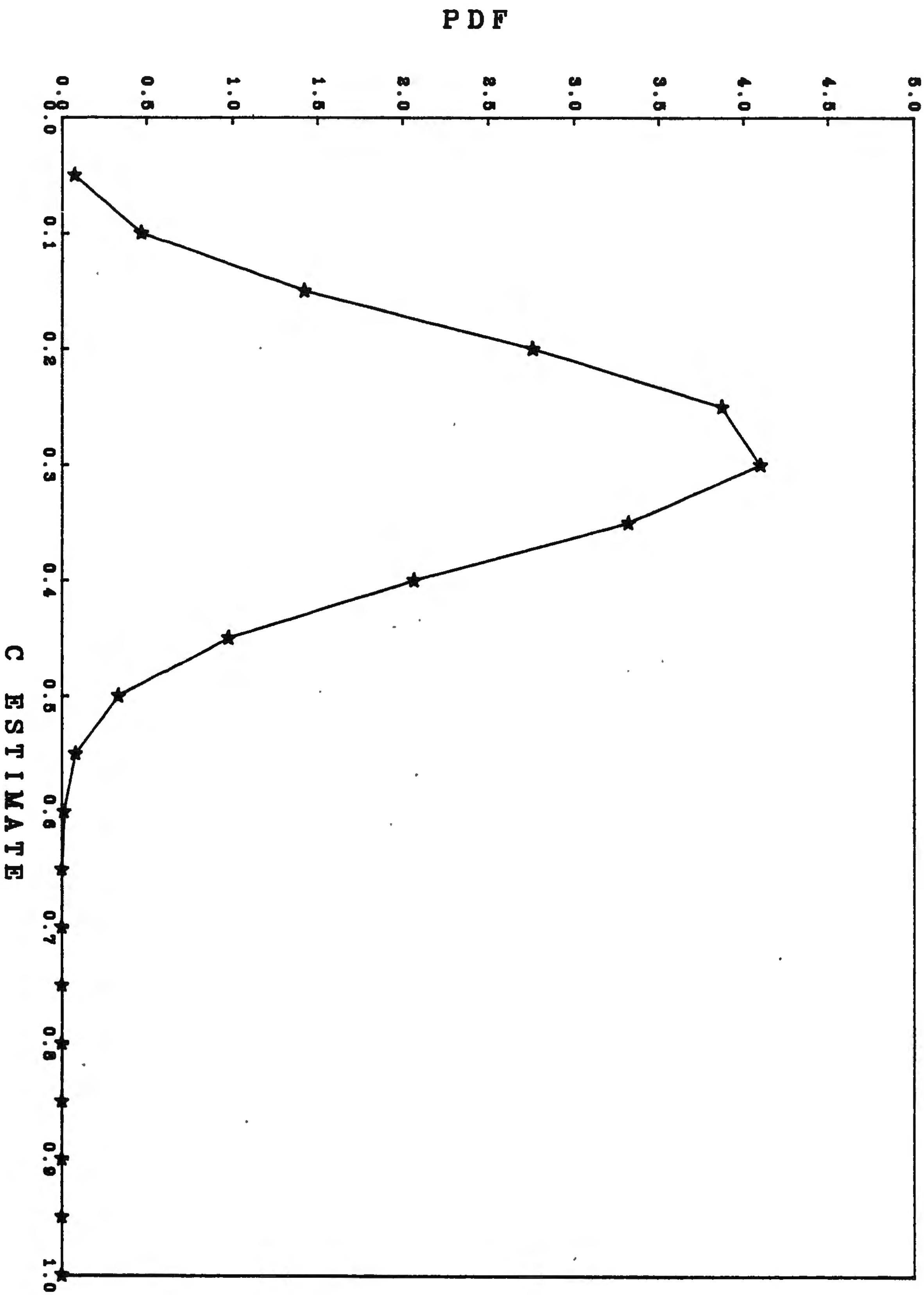
Table 1

Cumulative Distribution Function Calculation

Given : n = 32 and c = .3

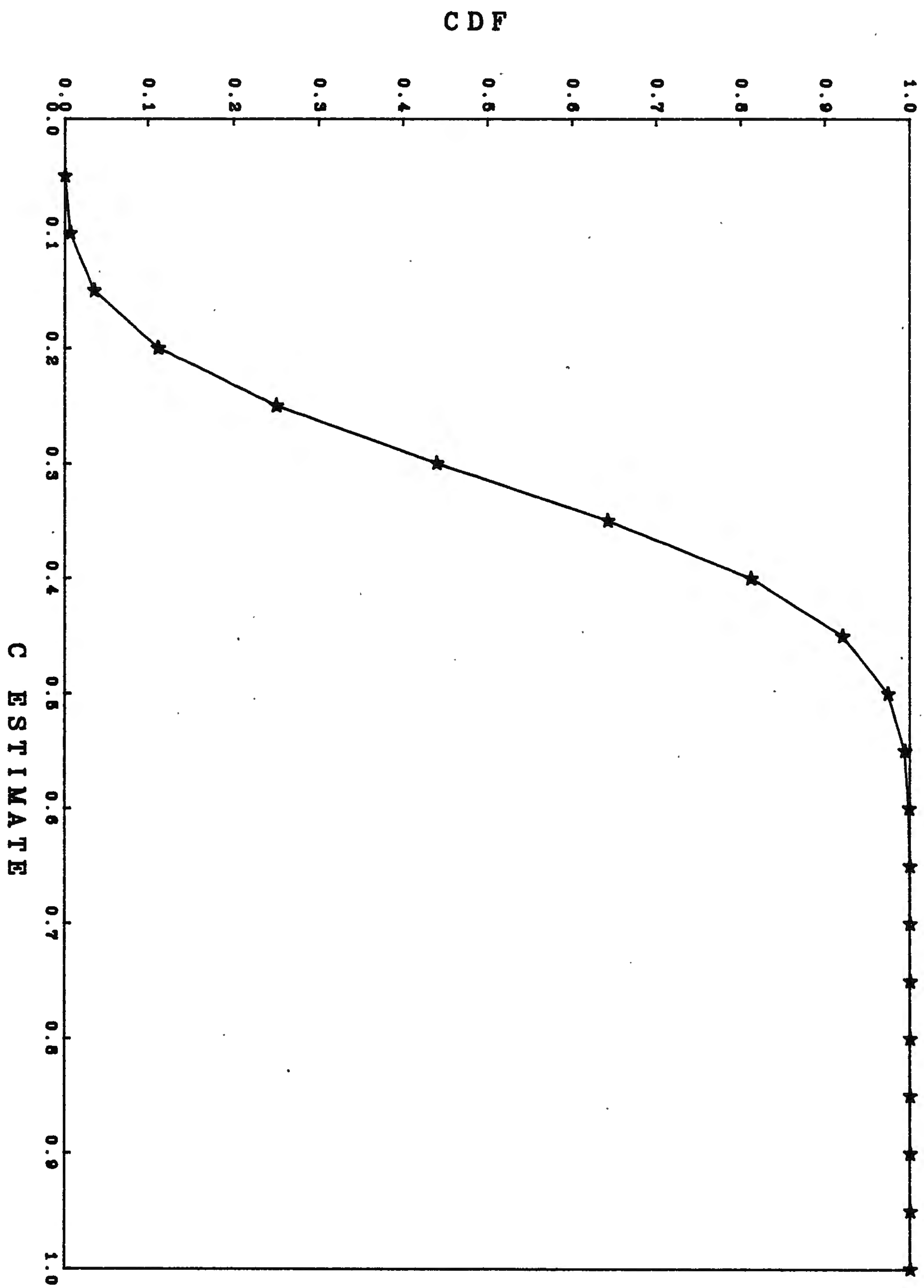
c estimate	CDF value
0.05000	0.00062
0.10000	0.00737
0.15000	0.03634
0.20000	0.11169
0.25000	0.24973
0.30000	0.43991
0.35000	0.64271
0.40000	0.81155
0.45000	0.92067
0.50000	0.97444
0.55000	0.99403
0.60000	0.99906
0.65000	0.99991
0.70000	0.99999
0.75000	1.00000
0.80000	1.00000
0.85000	1.00000
0.90000	1.00000
0.95000	1.00000
1.00000	1.00000

Table 2



PDF versus estimated coherence

Figure 1



CDF versus estimated coherence

Figure 2

IV. CONCLUSION

The magnitude-squared coherence function can be used for signal detection and passive sonar parameter estimation. To exploit this function it is necessary to estimate the quantity. The statistics of the estimate have been well documented, see reference (2) Carter, 1972; however, the computer software is complex and involves a number of subtleties because the hypergeometric function consists of combinations of very large and very small numbers in ways that preclude a simple straight-forward programming of the equations. Moreover, if you follow the procedure outlined in this memorandum, which on the surface is very straight-forward and use the computer software listed, then one can evaluate these complex mumerical problems with little or no numerical overflow or underflow problem in the parameter range of interest.

A computer program was written and tested to evaluate the probability density function and cumulative distribution function associated with the magnitude-squared coherence function.

This memorandum presents the formulas and FORTRAN programs tested for the particular case when the magnitude squared coherence equals .3 and the number of independent segments processed is 32.

The results of the sample test case are presented in tabular and graphical form in addition to the actual program listing.

REFERENCES

1. "Estimation of the Magnitude-Squared Coherence Function (Spectrum)," Carter, G.C., NUSC Technical Report, 19 May 1972.

2. "Handbook of Mathematical Functions," Abramowitz and Stegun, U.S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series.55, 1964.

3. "Receiver Operating Characteristics for a Linearly Thresholded Coherence Estimation Detector," Carter, G.C., IEEE Transactions on Acoustics, Speech, and Signal Processing, February 1977.

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